VISUAL MEASUREMENT SYSTEM (VMS)

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INTRODUCTION

In science and engineering, the ability to measure motion of an object with precision and accuracy is often one of the most important steps in evaluating the performance of a device or system. Measurement systems that achieve the level of accuracy required can be difficult and expensive to implement in scenarios where observed systems are small, and often cannot accommodate a separate measurement system. Additionally, introducing a measurement system can disrupt or alter the physical behavior of the observed system, leading to inaccurate results. In this whitepaper, we propose a Visual Measurement System (VMS) capable of measuring the position of multiple points in a 2-dimensional plane. We will also discuss verification of the Visual Measurement System by evaluating an example mechanical system.

METHODOLOGY

The Visual Measurement System itself is a MATLAB script capable of importing video files, deconstructing them into discrete frames, identifying objects in each image using K-mean clustering, and tracking objects position over time. The script then uses a numeric derivative method to evaluate the velocity and acceleration of the objects. To verify the accuracy of the Visual Measurement System, a simple observable system will be modeled mathematically and then physically constructed. The physical behavior of the system will be observed and analyzed by the visual measurement system and compared against the mathematical system model. To observe the mechanical system, an iPhone will be used to capture slow motion video at 240 Hz at 1080p resolution.

MECHANICAL SYSTEM SELECTION, MODELING, AND DESIGN

SYSTEM EVALUATION AND SELECTION

As candidates for a mechanical system, we will consider two simple examples: the simple pendulum and the mass-spring system. Both systems are canonically used as examples of systems that exhibit simple harmonic motion under reasonable assumptions. They are also well understood mathematically and can be modeled using ordinary differential equations. To observe a system, it is critical that the natural frequency of the system be significantly less than our sampling frequency. The sampling frequency in this case is the frame rate of our capture device, 240 Hz. For this reason, the system that can more readily attain a lower natural frequency will be more advantageous. The natural frequencies of the simple pendulum and mass-spring system are given by equations 1 and 2, respectively.

$$\omega_p = \sqrt{\frac{g}{l}} \tag{1}$$

$$\omega_m = \sqrt{\frac{k}{m}} \tag{2}$$

It should be noted that Equation 1 is only valid for small angles under the small angle approximation. By evaluating both systems' capable natural frequencies, a vertical mass-spring system was selected because we can more easily achieve lower frequencies than the simple pendulum. Additionally, the mass-spring system is more tunable as there are two parameters, mass m and spring rate k, that can be altered.

MASS-SPRING-DAMPER SYSTEM MODEL

To mathematically model the selected vertical mass-spring system, the system is conceptualized as a mass-spring-damper system. Experimentally, the damping coefficient c should be small for this system and is intended to represent the mechanical losses due to air resistance. The governing ODE for this system can be derived and represented as a function of position x and its time derivatives (Equation 3).

$$\ddot{x} = -\frac{k}{m}x - \frac{c}{m}\dot{x} \tag{3}$$

The ODE can be evaluated using MATLAB's *ode45* solver and constant coefficients to yield the state variables at a given time.

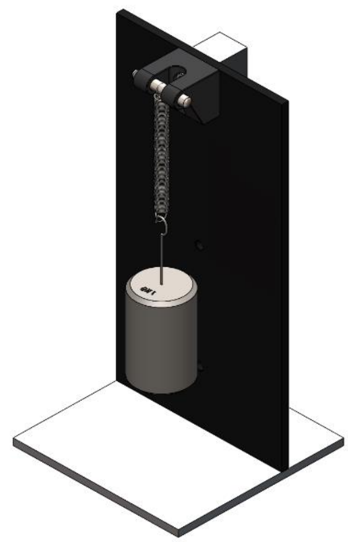

Figure 1: Mass-Spring System CAD Model

MECHANICAL SYSTEM DESIGN

The physical mass-spring system was designed so that an extension spring could be fixed at one end with a pin and a mass could easily be suspended from the other end. Additionally, the mass-spring system would have a monochromatic back plate to act as an easily recognizable background for the Visual Measurement System. The experimental fixture was also designed to be much stiffer than the extension spring so that our single spring model was representative of the physical system.

VISUAL MEASUREMENT SYSTEM

OVERVIEW

The Visual Measurement System workflow is shown in Figure 2.

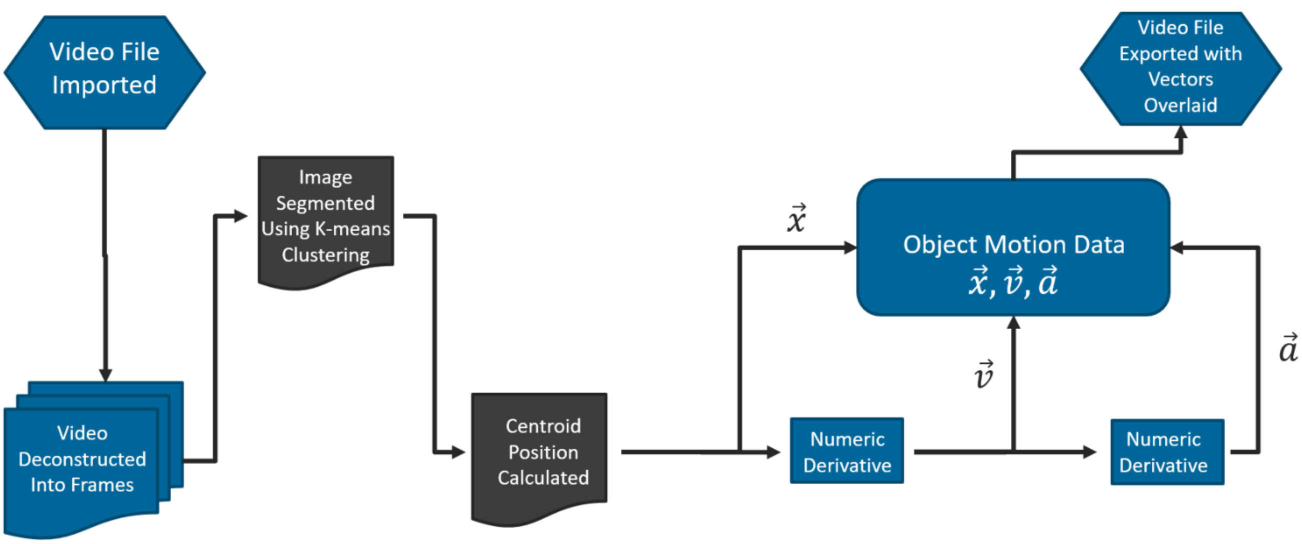

Figure 2: Visual Measurement Script Flowchart

K-MEANS CLUSTERING

K-means clustering is a popular technique used in image processing for segmenting an image into an integer number of clusters (K) based on color [1]. The technique examines each pixel of an image and uses an iterative scheme to find K RGB

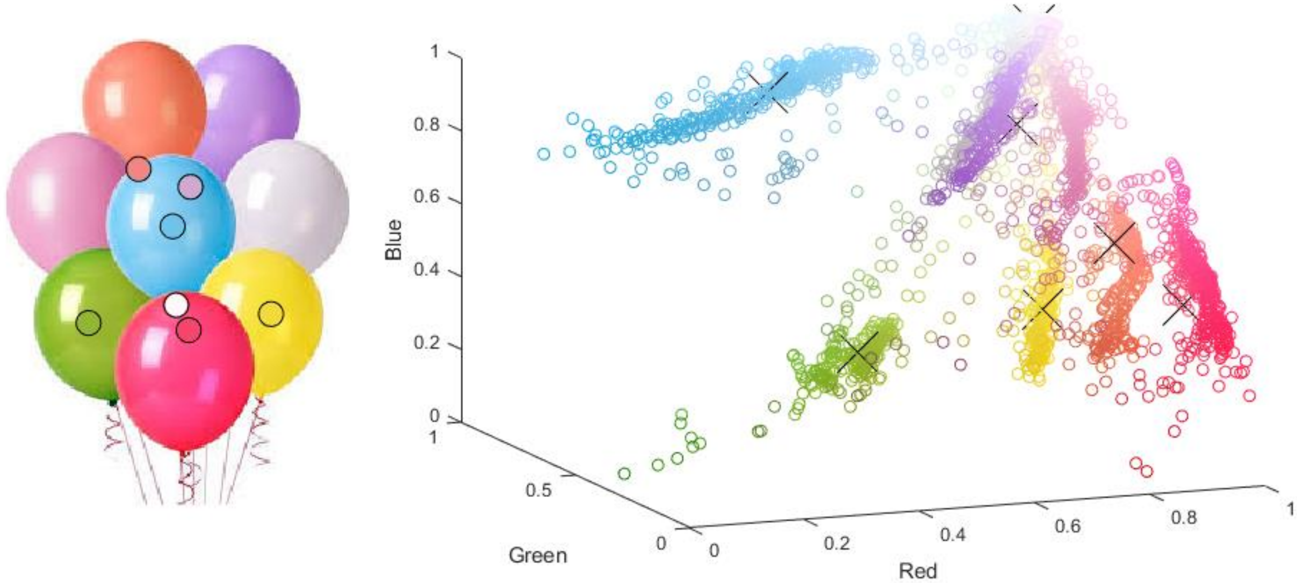

Figure 3: RGB Decomposition and Clustering of an Image with K = 7. The RGB Centroids are Marked with a Black 'X'.

centroids of color within an image. This technique is advantageous for our application because a colorful label can easily be used to mark the object of interest. One prerequisite to using this method is that the number of clusters must be known. After the RGB centroids are calculated, the image is broken down into clusters where each pixel belongs to the cluster centroid it is closest to in the RGB space. The geometric centroid of each cluster is then calculated. This process is then repeated for each frame in the video to yield the time series geometric centroid position data for each cluster.

NUMERIC DERIVATIVE

The derivative of each geometric centroid position is then taken to evaluate the velocity of each centroid. The numeric derivative method employed is a fourth-order central differencing method that is derived from a Taylor series expansion about point x [2].

$$x_i' = \frac{-x_{i+2} + 8x_{i+1} - 8x_{i-1} + x_{i-2}}{12\Delta t} \tag{4}$$

This method is advantageous because it has an error order of magnitude of Δt^4 and only requires 4 surrounding points. The first two and last two datapoints in the set are computed using equations 5 and 6 respectively. It should also be noted that equations 5 and 6 yield a larger error on the order of magnitude Δt^2.

$$x_i' = \frac{-3x_i + 4x_{i+1} - x_{i+2}}{2\Delta t} \tag{5}$$

$$x_i' = \frac{3x_i - 4x_{i-1} + x_{i-2}}{2\Delta t} \tag{6}$$

EXPERIMENTAL RESULTS AND VERIFICATION

SPRING RATE CHARACTERIZATION

To characterize spring rate, extension springs with a nominal spring rate of 120 N/m were tested in an Instron load frame with a 1 kN load cell. To calculate the spring rate, the springs were deflected from the equilibrium position to 25 mm. The resulting force versus displacement curve was analyzed by applying a linear curve fit to evaluate the spring rate as the slope of the curve. A total of six springs were characterized, which showed a consistent spring rate. The average spring rate recorded was 119.2 ± 1.66 N/m. The stiffness uncertainty was calculated by assuming a normal distribution of stiffnesses where the maximum variation is equal to ±4σ.

Figure 4: Mass Spring System with Colored Labels

MASS CHARACTERIZATION

A set of scientific masses was weighed to the nearest 10 mg on a precision balance to gather accurate data for the mass-spring system model. The mass selected had a nominal weight of 1 kg and was weighed precisely at 1.00029 kg. The 1 kg mass was selected to minimize the natural frequency of the mass spring system so that more data points could be captured per cycle. For our simulation we will use a mass value of 1 kg.

SYSTEM SIMULATION

To simulate the behavior of the mass-spring system, we evaluate the ODE in Equation 3 using MATLAB's *ode45* solver over the time domain [0, 10] seconds. The output over this timespan will replicate the physical system we will measure using the visual measurement system. The system will start when released from rest with a deflection of 15 mm in the positive direction. To approximate a damping constant, we examined the time the system takes to attenuate oscillation. Assuming the system reaches steady-state in 5τ, the damping constant was estimated at 0.01 N-s/m.

PHYSICAL SYSTEM MEASUREMENT

The physical mass-spring system was assembled and marked with several color-coded labels to indicate different bodies for the Visual Measurement System to track (Figure 4). Additionally, a second set of labels was placed on a static background to act as a reference point. To mimic the system simulation, a video recording for 10 seconds of oscillation was recorded and processed by the visual measurement system.

Parameter	Value
Spring Constant, *k*	119.2 [N/m]
Damping Constant, *c*	0.01 [N-s/m]
Mass, *m*	1.00 [kg]

SYSTEM ANALYSIS

To evaluate the validity of the visual measurement system, we compare the Visual Measurement System object motion data output to the system model simulated with constant coefficients and identical initial conditions. To process the Visual Measurement System data, we first need to scale the data length measurements from image pixels to meters. The colored labels in the image are 19.05 mm in diameter and are used for a reference to convert pixels to mm.

To analyze the frequency response of the system we can apply a Fast Fourier Transform (FFT) to our data to view the natural frequencies of the two systems. Figure 5 shows that in the time domain, the simulated system and physical systems show excellent agreement, with the main difference being that the systems are slightly out of phase. This could be due to a slight difference in initial conditions. The agreement is reinforced by analyzing the two systems in the frequency domain (Figure 6). The frequency magnitude plotted in Figure 6 has been normalized by the maximum magnitude. In the frequency domain, the physical and simulated systems also show excellent agreement and have a **0.9%** error when compared to the theoretical natural frequency. Additionally, both physical and simulated results fall within the acceptable natural frequency range considering the extension spring stiffness uncertainty.

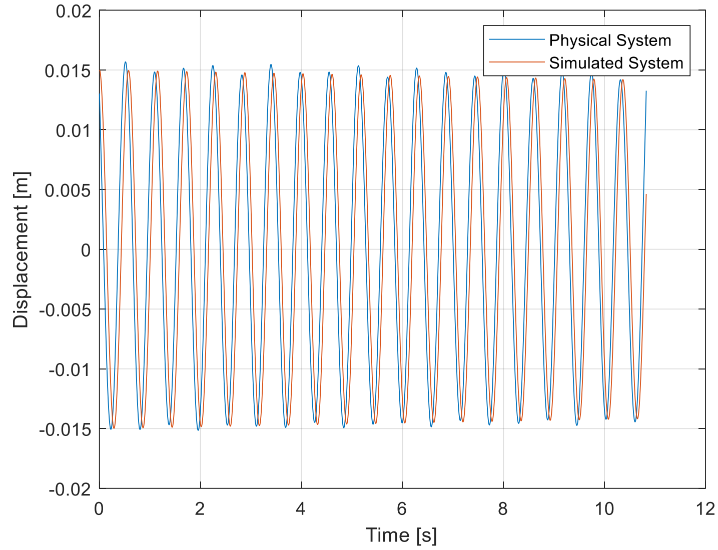

Figure 5: Time Domain Response of Physical and Simulated Mass Spring Systems

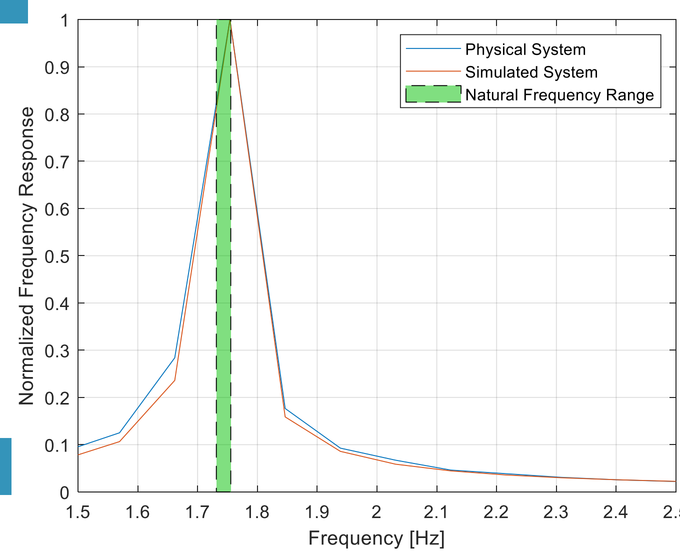

Figure 6: Frequency Domain Analysis of Physical and Simulated Mass Spring Systems

CONCLUSIONS

The Visual Measurement System was able to accurately record object motion data compared to an identical simulated mass-spring system when compared in the time domain. When compared in the frequency domain, the visual measurement system was able to identify the oscillating natural frequency of the mass spring system with 0.9% error compared to the theoretical natural frequency. The major advantages of the Visual Measurement System include its ease of setup, non-contact measurement, and support for multiple object tracking using image segmentation.

REFERENCES

[1] K. M. a. Y. J. C. Nameirakpam Dhanachandra, "Image Segmentation using K-means Clustering Algorithm and Subtractive Clustering Algorithm," *Procedia Computer Science,* vol. 54, pp. 764-761, 2015.

[2] F. B. Hildebrand, "Finite Difference Interpolation," in *Introduction to Numerical Analysis*, New York, Dover Publications, 1974, pp. 129 - 136.